Assessing the Feasibility of Generating a Uniform Magnetic Field with a Helmholtz Coil

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This report experimentally observes the magnetic field due to two parallel conducting coils, considering only the field at their centre. It is found that the field is uniform within 1 part in 1000 when the coils are separated by the same value as their radii, and that their radii must be the same for true uniformity. In this case, a Helmholtz coil is created. One application of this is in order to cancel out external magnetic fields, like the Earth's.

1. INTRODUCTION

Two conducting coils can be set up in a way as to generate a magnetic field inside the region of the coils. If this is done such that the magnetic field is uniform inside the region, the coils are together called a Helmholtz coil. This can be used for many applications, such as nullifying Earth's magnetic field when measurements of photo-electrons are made using magnetic fields of known strength [1].

A current through a wire induces a magnetic field $\vec{B}$ at a point in a direction $\hat{r}$ from a current of magnitude I. This is quantified by the Biot-Savart law,

$$\vec{B} = \frac{\mu_0}{4\pi} \int \frac{Id\vec{l} \times \hat{r}}{r^2}, \tag{1}$$

where $d\vec{l}$ is the vector in the direction of the current, r is the distance from the current to the point where the magnetic field density is measured, and μ_0 is the permeability of free space, a constant used in magnetic field considerations [2].

In this case, the Biot-Savart law is used to derive a specific relation for the magnetic field density magnitude $|\vec{B}|$ on the axis along the centre of two parallel current-carrying loops, by integration. $|\vec{B}|$ is related to: the current through both coils, I; the distance of both coils from their geometric centre, b; the radius of the first and second coil, a_1 and a_2, respectively; and the distance of the measurement to the centre of the system, x (taking the direction from the centre to the second coil as positive). The exact relation is,

$$|\vec{B}| = \frac{\mu_0 I a_1^2}{2((x+b)^2 + a_1^2)^{3/2}} + \frac{\mu_0 I a_2^2}{2((x-b)^2 + a_2^2)^{3/2}}. \tag{2}$$

Here we use an experimental approach to find the ideal coil separation, $2b$, in terms of the coil radii, a_1 and a_2, in order to generate the most uniform magnetic field possible.

2. METHODS

Two conducting wire coils were set up to be parallel within the plane of their area on stands along a measuring track. They were of a 'single winding', i.e. they were a single wire. They were connected to a variable power supply in series so that the current would be the same through both coils, and this was set to around 10 A and measured. A Hall probe was mounted on the track, and centred in the geometric centre of the coil areas. It was able to move along the axis of both coils, and a distance measurement could be taken from the measuring track. A zero position was taken where the end of the probe was in the centre of the first coil, and so measurements were taken against this. A range of distances of 2 radii either side of each coil was taken, with separation of 5 mm.

The power supply was switched on only when measurements were being taken, so that there was not a heating effect in the connected wires, which would change the magnetic field of the system over time. Both coils were firmly fixed in position while measurements were made, and it was made sure that the Hall probe could only move along the axis of the coils, and did not move vertically or horizontally. The measurements of magnetic field density were taken by 'freezing' the Hall probe, which stopped the measurement on a value, since it was fluctuating. The uncertainties and calculations used for both distance and magnetic field density are detailed in the errors appendix.

3. RESULTS

Figure 1 shows the magnetic field density distribution along the axis of two wire coils at optimal separation for a uniform magnetic field in the centre of the coils. A point at x = 90 mm was tested against the fit and deemed to be an outlier by Chauvenet's criterion [3], and so was removed. In the horizontal normalised residuals plot (right), the values for a magnetic field of higher than 200 μT (in red) were omitted from the histogram due to the fact that the curve flattens off so the points were many standard errors from the curve in some points, which was not representative of the true fit. The calculated coil separation was 40.8 mm and the calculated coil radii from left to right were 41.4 mm and 40.1 mm.

Figure 2 shows the same setup but with varying coil spacings, resulting in a non-uniform magnetic field in the centre of the two coils. The calculated separations were, from smallest to largest, 29.4 mm, 39.2 mm, and 48.0 mm.

Figure 3 aims to show the change in magnetic field with respect to distance along the central axis of the coils. Note that the gradient curve is almost flat in the centre of the figure (at x = 0), but not equal to 0.

In all figures, the parameters (the current through the coils, the radii of the coils, and the coil separation) for the fitted curve are found by χ^2 minimisation [3].

4. DISCUSSION

In figure 1, the residual plots show that the experimental data fits with the theoretical curve well, as both the vertical and horizontal histograms fit a normal curve (plotted). The vertical errors (bottom) fit less well, as points on the his-

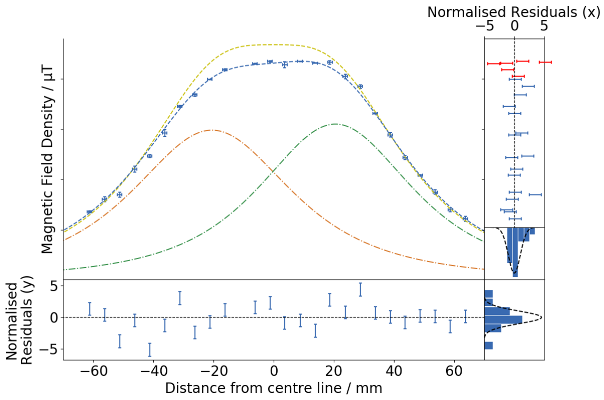

FIG. 1: The orange and green lines are the magnetic field contributions from two wire coils. The blue line is their algebraic sum. The yellow line shows the unfitted curve. Plotted are the measured values of magnetic field density along a line through the centre of both coils.

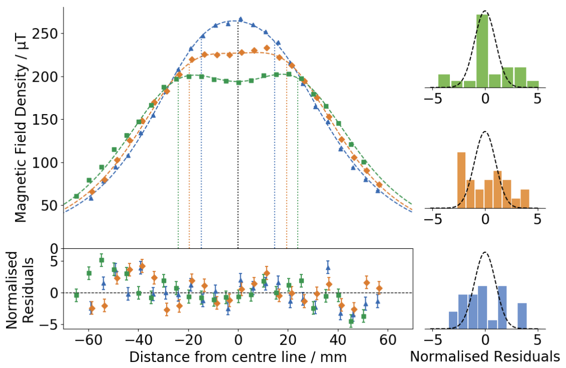

FIG. 2: A graph showing the magnetic field due to two wire coils, spaced differing distances apart. Vertical lines show the respective placements of the coils. Plotted are the measured values, with residuals from the vertical error bars below and histograms of these to the right.

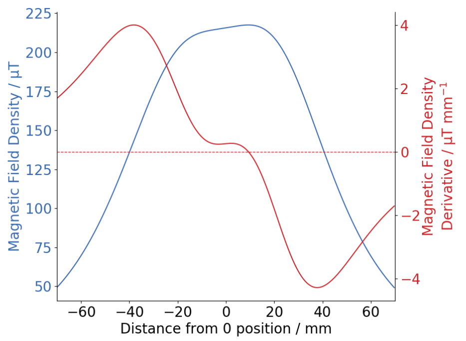

FIG. 3: A plot showing the data from figure 1 with its gradient plotted on the same axes.

togram can be seen well above the normal curve. This may be because the uncertainty on the magnetic field (B field) measurement was underestimated in some cases. This may be the result of random variation and more B field measurements should have been taken per point. It should not be due to a systematic error in the probe since this would simply translate the plot up or down, resulting in a different curve fit.

The fitted curve in figure 1 (blue) is lower intensity than the unfitted curve (yellow). This results in an underestimation of the current, and an overestimation of both coil radii, and the coil separation. These fits of the radii, separation, and current may not be representative of the true values due to a systematic error in the B field measurement. This could be that the Hall probe was not centralised or the coils were not at the same level or in the same direction. All of these could mean the vector of magnetic field density was not perpendicular to the Hall probe sensor, so it measured a fraction of the intensity of the magnetic field. It is also proposed that there was a systematic error due to an external magnetic field due to larger magnetic equipment in the same building as the laboratory. However these may not be the case as the unfitted curve intersects the measured values at large values of x, so a systematic error is not visible here. The discrepancy could be due to the calibration curve of the Hall probe not being set correctly, so at larger values of magnetic field density, it measured a worse value than the real value.

Figure 2 shows the variation of the curve shape with different spacings of the coils. Observe the difference in the uniformity of the region within the coils. It is seen that the most visibly uniform of the three curves is the orange curve, where the separation of the coils is closest to the radius of the coils.

Figure 3 shows the curve for a separation of one radius, and the relative gradient of this curve. See that in the centre, the gradient is nearly zero (around 0.25 μT mm^{-1}). This means that over a range of -20 to 20 mm the field changes by only around 5 μT. For a field of 210 μT this is fairly negligible, and the rate of change of magnetic field is only 0.1% of its strength, showing that it is feasible to use conducting wire coils to create a uniform magnetic field.

5. CONCLUSIONS

The distribution of magnetic field density was measured inside and outside a pair of conducting coils in order to determine the ideal separation in order to generate a uniform magnetic field inside the coils. It was found that the ideal separation was equal to the average radius of the coils, where the rate of change of magnetic field density was 0.1% of the magnetic field density within the coils. This is the separation at which the apparatus could be called a Helmholtz coil, as it produced the most uniform magnetic field density distribution available. This shows that it would be feasible to nullify the Earth's magnetic field within a region using a Helmholtz coil at the correct orientation.

In this particular case the coils were of different radii so the most uniform magnetic field inside the region was the situation with the lowest rate of change of magnetic field density.

References

[1] A. E. Ruark and M. F. Peters, *Helmholtz Coils for Producing Uniform Magnetic Fields*, J. Opt. Soc. Am., 13, 205-212 (1926).

[2] H. D. Young and R. A. Freedman, *University Physics with Modern Physics*, 13th Ed., Pearson Addison-Wesley, San Francisco (2012).

[3] I. G. Hughes and T. P. A. Hase, *Measurements and Their Uncertainties*, Oxford University Press: Oxford (2010).

APPENDIX A: ERRORS APPENDIX

The standard errors on all measurements of distance were taken to be half an analogue division of the measuring device used. Whenever distances were combined, such as when the zero distance was taken from the measured distance, the errors were added in quadrature in accordance with the equation,

$$\alpha_d = \sqrt{\alpha_{d_1}^2 + \alpha_{d_2}^2}, \tag{A1}$$

where α_{d_1} and α_{d_2} are the uncertainties on the distance measurements d_1 and d_2, respectively, and α_d is the final uncertainty on the final distance measurement d. [This equation, like all of the equations included in Appendix A, is based on the error analysis formula given in I. G. Hughes and T. P. A. Hase, *Measurements and Their Uncertainties*, Oxford University Press: Oxford (2010).]

The variation on the reading on the Gauss Meter was accounted for by taking a number, N, of measurements and calculating the mean and standard error of these measurements. The mean is calculated using the equation,

$$\bar{B} = \frac{1}{N}\sum_{i=1}^{N} B_i, \tag{A2}$$

where $\bar{B}$ is the mean measurement of B and B_i are individual measurements of the magnetic field density B.

The sample standard deviation, σ_{sample}, of the set of measurements is worked out using the equation,

$$\sigma_{sample} = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N} d_i^2}, \tag{A3}$$

where $d_i = \bar{B} - B_i$. The uncertainty in the measurement of $\bar{B}$ is taken to be its standard error, α_B, where

$$\alpha_B = \frac{\sigma_{sample}}{\sqrt{N}}. \tag{A4}$$

In this case, six measurements were taken for every measurement of position.